Mixed Team Competition 2006

Instructions: Do as many problems as possible, and write your answers on the answer sheet provided. You may not use a calculator. Be sure to work as a TEAM and Have Fun!

1. In a certain flower garden, each flower was either red, yellow, or blue, and all three colors were represented. A statistician once visited the garden and made the observation that whatever three flowers you picked, at least one of them was bound to be red. A second statistician visited the garden and made the observation that whatever three flowers you picked, at least one was bound to be yellow.

Two students heard about this and got into an argument. The first student said: “It therefore follows that whatever three flowers you pick, at least one is bound to be blue, doesn’t it?” The second student said: “Of course not!” Which student was right?

2. The following question is supposedly used in Microsoft job interviews.

“There are four people who need to cross a river at night. There is a bridge that can only hold up to two people at a time. There is one flashlight that must be used when crossing. (It is extremely dark, and someone must bring the flashlight back to the others; no throwing anything, no halfway crosses, etc.) The four people take different amounts of time to cross the river. If two people cross together, they travel at the slower person’s rate. The times are 10 minutes, 5 minutes, 2 minutes, and 1 minute for each of the four individuals. How fast can they complete the passage? Can they do it in 19 minutes? In 17 minutes?

3. Everyone knows that February is the shortest month of the year. What is the longest month of the year in Ohio?
4. There is a certain club called the Barber’s Club. The following facts are known about it.

(a) Every member of the club has shaved at least one member.
(b) No member has ever shaved himself.
(c) No member has ever been shaved by more than one member.
(d) There is one member who has never been shaved at all.

The number of members of this club has been kept a strict secret. One rumor has it that there are less than a thousand members. Another rumor has it that there are more than a thousand members. Which of the two rumors is true? i.e., How many members are in the club?

5. Another Barber’s club follows the following conditions.

(a) If any member has shaved any member - whether himself or another - then all members have shaved the shaver, although not necessarily at the same time.
(b) Four of the members are named Guido, Lorenzo, Petruchio, and Cesare.
(c) Guido has shaved Cesare.

Has Petruchio shaved Lorenzo or not?

6. In Dunrovia they have a curious coinage system which includes $4 and $7 coins. Can you determine the smallest amount of money that can be made up in two different ways, using only $4 and $7 coins? At least one of each type of coin must be used.

7. Using the preceding problem, can you determine the smallest amount of money that can be made up in three different ways, using only $4 and $7 coins? At least one of each type of coin must be used.

8. The integer 66 can be written as the sum of two smaller integers. One integer is 3 more than twice the other integer. Find the larger of the two integers.
9. The following questions relate to the inhabitants of the Island of Knights and Knaves, where knights always tell the truth and knaves always lie.

(a) Is it possible for any inhabitant of this island to claim that he is knave?

(b) Is it possible for an inhabitant of the island to claim that he and his brother are both knaves?

(c) Suppose that inhabitant $A$ says about himself and his brother $B$: “At least one of us is a knave.” What type is $A$ and what type is $B$?

(d) Suppose instead that $A$ says: “Exactly one of us is a knave.” What type is $B$?

10. Inspector Craig of Scotland Yard was called to the Island of Knights and Knaves to help find a criminal named Arthur York. What made the process difficult was that it was not known whether Arthur York was a knight or a knave.

(a) THE FIRST TRIAL:
One suspect was arrested and brought to trial. Inspector Craig was the presiding judge. Here is a transcript of the trial.

CRAIG: What do you know about Arthur York?

DEFENDANT: Arthur York once claimed that I was a knave.

CRAIG: Are you by any chance Arthur York?

DEFENDANT: Yes.

Is the defendant Arthur York?
(b) THE SECOND TRIAL:
Another suspect was arrested and brought to trial. Here is a transcript of the trial.

CRAIG: The last suspect was a strange one! He actually claimed that he was Arthur York. Did you ever claim to be Arthur York?

DEFENDANT: No.

CRAIG: Did you ever claim that you were not Arthur York?

DEFENDANT: Yes.

Is the defendant Arthur York?

(c) THE THIRD TRIAL:
“Don’t despair,” said Craig to the chief of the island police. “We may find our man yet!”

Well a third suspect was arrested and brought to trial. He brought with him his defense attorney, and the two made the following statements in court.

DEFENSE ATTORNEY: My client is indeed a knave, but he is not Arthur York.

DEFENDANT: My attorney always tells the truth!

Is the defendant Arthur York?
SUDOKU!

11. A Sudoku puzzle consists of a nine by nine square grid subdivided into nine three by three boxes. Some of the squares contain numbers. The object is to fill in the remaining squares so that every row, every column, and every three by three box contains each of the numbers one through nine exactly once. Solving a Sudoku puzzle involves pure logic. No guesswork is needed or even desirable. Be careful not to repeat a number where you shouldn’t, because a wrong answer may force you to start over. Try to solve the following Sudoku puzzle. Have fun!