```
> restart:with(student):
```

I ex1

```
> Sum('(-1)^k*ln(k)/(k^3)', 'k'=1..infinity)=
  sum('(-1)^k*ln(k)/(k^3)', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k \ln(k)}{k^3} = \sum_{k=1}^{\infty} \frac{(-1)^k \ln(k)}{k^3}$$

ex2

```
> Sum('1/(k^k)', 'k'=1..infinity)= sum('1/(k^k)', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{1}{k^k} = \sum_{k=1}^{\infty} \frac{1}{k^k}$$

ex3

```
> Sum('1/(2^k)', 'k'=1..infinity)= sum('1/(2^k)', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{1}{2^k} = 1$$

ex4

```
> Sum('1/ln(k)', 'k'=2..infinity)= sum('1/ln(k)', 'k'=2..infinity);
>
```

$$\sum_{k=2}^{\infty} \frac{1}{\ln(k)} = \sum_{k=2}^{\infty} \frac{1}{\ln(k)}$$

II ex1

```
> Sum('k^2 *x^k', 'k'=1..infinity)= sum('k^2*x^k', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} k^2 x^k = -\frac{x(x+1)}{(x-1)^3}$$

ex2

```
> Sum('x^k/(k^2)', 'k'=1..infinity)= sum('x^k/k^2',
  'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{x^k}{k^2} = \operatorname{polylog}(2, x)$$

ex3

```
> Sum('(-1)^k*(x-3)^k/(3^k)', 'k'=1..infinity)=
  sum('(-1)^k*(x-3)^k/(3^k)', 'k'=1..infinity);
```

III ex1

$$\sum_{k=1}^{\infty} \frac{(-1)^k (x-3)^k}{3^k} = 3\,\frac{-\frac{1}{3}x+1}{x}$$

```
> taylor( cos(x), x=0, 12 );
```

$$1-\frac{1}{2}x^2+\frac{1}{24}x^4-\frac{1}{720}x^6+\frac{1}{40320}x^8-\frac{1}{3628800}x^{10}+O(x^{12})$$

III ex2

```
> taylor( sin(x)/x, x=0, 12 );
```

$$1-\frac{1}{6}x^2+\frac{1}{120}x^4-\frac{1}{5040}x^6+\frac{1}{362880}x^8-\frac{1}{39916800}x^{10}+O(x^{11})$$

III ex3

```
> taylor( ln(x), x=1, 12 );
```

$$x-1-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\frac{1}{5}(x-1)^5-\frac{1}{6}(x-1)^6+\frac{1}{7}(x-1)^7-\frac{1}{8}(x-1)^8+\frac{1}{9}(x-1)^9-\frac{1}{10}(x-1)^{10}+\frac{1}{11}(x-1)^{11}+O((x-1)^{12})$$

```
>
```

I ex 1

```
> Sum('(-1)^k*(2*k+1)/(5*k+7)', 'k'=1..infinity)=
  sum('(-1)^k*(2*k+1)/(5*k+7)', 'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty}\frac{(-1)^k(2k+1)}{5k+7}=\mathit{undefined}$$

I ex 2

```
> Sum('(-1)^k*(2^k)/(k!)', 'k'=1..infinity)=
  sum('(-1)^k*(2^k)/(k!)', 'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty}\frac{(-1)^k 2^k}{k!}=\mathbf{e}^{(-2)}(1-\mathbf{e}^2)$$

I ex 3

```
> Sum('(-1)^k*ln(k)/k', 'k'=1..infinity)= sum('(-1)^k*ln(k)/k',
  'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty}\frac{(-1)^k\ln(k)}{k}=\sum_{k=1}^{\infty}\frac{(-1)^k\ln(k)}{k}$$

I ex 4

```
> Sum('(-1)^k*(k+1)/k^2', 'k'=1..infinity)= sum('(-1)^k*(k+1)/k^2',
  'k'=1..infinity);
>
>
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k (k+1)}{k^2} = -2 \text{ hypergeom}([1, 1, 1], [2, 2], -1) + \frac{1}{4} \text{ hypergeom}([2, 2, 2], [3, 3], -1)$$

I ex 5

```
> Sum('(-1)^k*(k!)*(k!)/((2*k)!)', 'k'=1..infinity)=
  sum('(-1)^k*(k!)*(k!)/((2*k)!)', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k (k!)^2}{(2\,k)!} = -\frac{1}{2} \text{ hypergeom}\left([1, 2], \left[\frac{3}{2}\right], \frac{-1}{4}\right)$$

```
>
```

I ex 1

```
> Sum('(-1)^k*(2^k)*(x^k)/(k^2)', 'k'=1..infinity)=
  sum('(-1)^k*(2^k)*(x^k)/(k^2)', 'k'=1..infinity);
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k 2^k x^k}{k^2} = -2\,x \text{ hypergeom}([1, 1, 1], [2, 2], -2\,x)$$

I ex2

```
> Sum('(-1)^k*(x^k)/(k^2)', 'k'=1..infinity)=
  sum('(-1)^k*(x^k)/(k^2)', 'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k^2} = -x \text{ hypergeom}([1, 1, 1], [2, 2], -x)$$

II ex1

```
> series(1/(2+x^2), x=0);
```

$$\frac{1}{2} - \frac{1}{4}x^2 + \frac{1}{8}x^4 + O(x^6)$$

III ex1

```
> taylor( cos(x), x=pi/2, 6 );
```

$$\cos\left(\frac{1}{2}\pi\right) - \sin\left(\frac{1}{2}\pi\right)\left(x - \frac{1}{2}\pi\right) - \frac{1}{2}\cos\left(\frac{1}{2}\pi\right)\left(x - \frac{1}{2}\pi\right)^2 + \frac{1}{6}\sin\left(\frac{1}{2}\pi\right)\left(x - \frac{1}{2}\pi\right)^3 + \frac{1}{24}\cos\left(\frac{1}{2}\pi\right)\left(x - \frac{1}{2}\pi\right)^4 - \frac{1}{120}\sin\left(\frac{1}{2}\pi\right)\left(x - \frac{1}{2}\pi\right)^5 + O\left(\left(x - \frac{1}{2}\pi\right)^6\right)$$

III ex3

I ex2

```
> Sum('(-1)^k*(2*x-1)^k/(k^3)', 'k'=1..infinity)=
  sum('(-1)^k*(2*x-1)^k/(k^3)', 'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k (2\,x-1)^k}{k^3} = (-2\,x+1) \text{ hypergeom}([1, 1, 1, 1], [2, 2, 2], -2\,x+1)$$

I ex4

```
> Sum('(-1)^k*x^(2*k+1)/(k!*(k+1)!*2^(2*k+1))', 'k'=1..infinity)=
  sum('(-1)^k*x^(2*k+1)/(k!*(k+1)!*2^(2*k+1))', 'k'=1..infinity);
>
```

$$\sum_{k=1}^{\infty} \frac{(-1)^k x^{(2k+1)}}{k!\,(k+1)!\,2^{(2k+1)}} = -\frac{1}{16} x^3 \operatorname{hypergeom}\left([1], [2, 3], -\frac{1}{4} x^2\right)$$

I ex2

```
> Int(1/(1+x^3),x)=int(series(1/(1+x^3),x=0),x);
```

$$\int \frac{1}{1+x^3}\,dx = x - \frac{1}{4} x^4 + O(x^7)$$

I ex2

```
> taylor( exp(x), x=1, 6 );
```

$$\mathbf{e} + \mathbf{e}\,(x-1) + \frac{1}{2}\mathbf{e}\,(x-1)^2 + \frac{1}{6}\mathbf{e}\,(x-1)^3 + \frac{1}{24}\mathbf{e}\,(x-1)^4 + \frac{1}{120}\mathbf{e}\,(x-1)^5 + O((x-1)^6)$$

```
>
```