

Challenge of Champions Test 2006

1. The sides of a triangle are in the ratio of 2 : 3 : 4. Find the cosine of the smallest angle.

2. Find all solutions to

$$x^{\log x} = \frac{x^3}{100}$$

where $\log x = \log_{10} x$.

3. Let p, q and r be integers such that

$$f(x) = x^4 + 4x^3 + 6px^2 + 4qx + r$$

is divisible by $x^3 + 3x^2 + 9x + 3$. Find $p + q + r$.

4. If $\sin x + \cos x = -1/5$ and $3\pi/4 \leq x \leq \pi$, find $\cos(2x)$.

5. Show that

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdots - \frac{1}{2005} + \frac{1}{2006} > \frac{1}{5}.$$

6. Let $f(x) = ax^2 + bx + c$ where a, b and c are integers and 5 divides $f(n)$ for any integer n . Show that a, b and c are each divisible by 5.

7. The real numbers a, b and c are such that $a + b + c = 5$ and

$$\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{2}{3}.$$

Find the value of

$$\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a}.$$

8. If a tetrahedron (a solid with 4 equilateral triangles as faces) has edge length 1 for each edge. What is the height of the tetrahedron?
9. A grasshopper jumps along a number line starting at the point 0. The first jump takes him 1 cm, the second 2 cm, the third 3 cm and so on. Each jump takes him to the right or to the left. Can the grasshopper return to the point 0 on the 73rd jump?
10. Can one find 2006 positive integers whose sum is equal to its product?