Mixed Team Competition 2004

Instructions: Do as many problems as possible, and write your answers on the answer sheet provided. You may not use a calculator. Be sure to work as a **TEAM** and **Have Fun**!

1. The price of a silver bracelet is originally \$80.00. If the price is decreased by 25 percent, and then increased by 25 percent, what is the resulting price?

2. The number 42 can be written in three different ways as the sum of two or more consecutive positive numbers:

$$42 = 13 + 14 + 15 = 9 + 10 + 11 + 12 = 3 + 4 + 5 + 6 + 7 + 8 + 9.$$

   How many different ways can you write 105 as the sum of two or more consecutive positive numbers?

3. A duck swims 8 miles downstream in 2 hours. The duck returns upstream in 4 hours. How fast does the duck swim in the water?

4. Among those taking part in an election, the proportion of men to women was 17:15. Had 90 fewer men and 80 fewer women taken part the proportion would have been 8:7. How many people took part in the election?

5. Find the absolute value of the sum of all values of x that must be excluded from the domain of

$$f(x) = \frac{\frac{2}{2+x}}{2 - \frac{2}{2+x}}.$$

6. John sells an item at \$15.00 less than the list price and receives 10 percent of his selling price as the commission. Andrea sells the same item at \$25.00 less than the list price and receives 20 percent as the commission. If they both got the same commission, then what is the list price?

7. How many integers m satisfy $\left|\frac{m}{3} - 4\right| \leq 3$?

8. The integer 66 can be written as the sum of two smaller integers. One integer is 3 more than twice the other integer. Find the larger of the two integers.

9. Jane has \$3.08 in pennies, nickels, and quarters. She has 4 more pennies than quarters, and one more nickel than pennies. How many nickels does she have?

10. Someone said that the proportion women employees in the office is more than 60% but less than 65%. What is the minimum number of employees in the office?

11. In a cross country run, Steve placed exactly in the middle among all participants. Dan placed lower, in the tenth place, and Larry placed sixteenth. How many runners participated in the race?

12. Determine $\sin\left(\arcsin\left(\frac{3}{5}\right) - \arcsin\left(\frac{5}{13}\right)\right)$. Note: arcsin is the inverse sine.

13. The equation $(x+1)(x+2)(x+3)(x+4)+1=0$ has all real roots. Find the product of these roots.

14. Four frogs sit in a row. Every five seconds two neighboring frogs hop into each other's places. Sometime after 80 seconds, but before 100 seconds, the frogs are seen to be in their original order. On which jump did this occur?

15. Let n be the smallest positive integer such that

$$\sqrt{n} - \sqrt{n-1} < 0.1.$$

Find the greatest integer that is less than or equal to $\sqrt{n}$.

16. For how many values of x in the interval $[0, 2\pi]$ is it true that

$$\sin(125x+3) = \cos(125x+3)?$$

17. Suppose that

$$(1-2x+3x^2)^7 = a_0 + a_1 x + ... + a_{14} x^{14},$$

Find $a_0 + a_1 + ... + a_{14}$.

18. What is the highest power of 2 that divides $3^{3^{100}} - 1$?

19. Let a_1, a_2, a_3... be the sequence defined by

$$a_1 = \sqrt[3]{3}, \quad \text{and} \quad a_n = (a_{n-1})^{\sqrt[3]{3}} \quad \text{for each } n \geq 2.$$

Find the smallest integer n for which a_n is an integer.

20. Suppose that (x, y) lies on both

$$|x| + x + y = 8 \quad \text{and} \quad x + |y| - y = 11.$$

What is $x + y$?