

## Mixed Team Competition 2003

Instructions: Do as many problems as possible, and write your answers on the answer sheet provided. You will not receive any credit for “yes” or “no” answers given with no explanation. You may not use a calculator. Be sure to work as a **TEAM** and **Have Fun!**

1. Among those taking part in an election, the proportion of men to women was 17:15. Had 90 fewer men and 80 fewer women taken part the proportion would have been 8:7. Does this determine how many people took part in the election? If so, how many people took part in the election?
2. Four frogs sit in a row. Every five seconds two neighboring frogs hop into each other's places. Sometime after 80 seconds, but before 100 seconds, the frogs are seen to be in their original order. Is it possible to determine on which jump that occurred? Explain.
3. The number 42 can be written in three different ways as the sum of two or more consecutive positive numbers:

$$42 = 13 + 14 + 15 = 9 + 10 + 11 + 12 = 3 + 4 + 5 + 6 + 7 + 8 + 9.$$

How many different ways can you write 105 as the sum of two or more consecutive positive numbers? List them.

4. A sale item was marked down by the same percentage for three years in a row. After two years, the item was 51% off the original price. What was the price (in terms of the original price) after three years?
5. In Dunrovia they have a curious coinage system which includes \$4 and \$7 coins. Can you determine the smallest amount of money that can be made up in two different ways, using only \$4 and \$7 coins? At least one of each type of coin must be used.
6. Using the preceding problem, can you determine the smallest amount of money that can be made up in three different ways, using only \$4 and \$7 coins? At least one of each type of coin must be used.
7. My bathroom scale is set incorrectly but otherwise it works fine. It shows 10 kilograms when Steve stands on it, 14 kilograms when Maria stands on it, but 22.5 kilograms when Steve and Maria both stand on it. Is the scale set too high or too low? Explain.
8. Someone said that the proportion women employees in the office is more than 60% but less than 65%. What is the minimum number of employees in the office? Explain.
9. Is it possible to write the numbers 1 to 100 along a line, so that the difference between any two adjacent numbers is at least 50? Explain.

10. Is it possible to write the numbers 1 to 100 around a circle, so that the difference between any two adjacent numbers is at least 50? Explain.
11. Two communities live on a remote island. In the one community, everyone has green hair, and in the other, everyone has blue hair. In the Lonely Planet's Tourist Guide it says that those in one of the communities always tell the truth, while those in the other community always tell lies. On my visit to the island I met with two representatives from the island, one from each community. They had their heads covered with scarves so that I could not see their hair color. I asked each one of them the same question: Is your hair green? They both answered "no." From this information, can you determine which community tells the truth? Explain.
12. At another time on the same island (see the preceding problem) I overheard the following quarrel between four residents  $A, B, C$  and  $D$ , "You are a liar!" shouted  $B$  to  $A$ . "It is you who is a liar!" exclaimed  $C$  to  $B$ . "They're both liars," interjected  $D$  to  $C$ , "and furthermore, you are as well!" I could not see their hair but from this little dispute I was able to deduce the color of each person's hair. Which one(s) has/have green hair?
13. Margaret and Bruce start counting at the same time and the same speed. Margaret counts forward by 3's starting with 70:

70, 73, 76, . . . ,

while Bruce counts backwards by 7's from 1996:

1996, 1989, 1982, . . .

Will there be two numbers that they will count simultaneously that are less than 4 apart? Explain.

14. A number is placed in every square of a  $3 \times 3$  grid. The sum of the numbers in each row and each column and each diagonal is 33. Is the number in the center square determined? Explain.
15. Can you divide 13 presents, of values 1, 2, . . . 13 dollars among Alice, Bertha, and Carol so that each receive the same total value?
16. Can you divide 14 presents, of values 1, 2, . . . 14 dollars among Alice, Bertha, and Carol so that each receive the same total value?
17. In a cross country run, Steve placed exactly in the middle among all participants. Dan placed lower, in the tenth place, and Larry placed sixteenth. How many runners participated in the race?

18. In the middle of the climatic episode of J.J. Rowling's children's novel *Harry Potter and the Scorer's Stone*, Harry Potter and Hermione Granger find themselves in a room whose two exits are blocked by raging flames. On a table in the room stand seven bottles, along with a piece of paper inscribed with the following verse:

Danger lies before you, while safety lies behind,  
Two of us will help you, whichever you would find,  
One among us seven will let you move ahead,  
Another will transport the drinker back instead,  
Two among our number hold only nettle wine,  
Three of us are killers, waiting hidden in line.  
Choose, unless you wish to stay here forevermore,  
To help you in your choice, we give you these clues four:

First, however slyly the poison tries to hide  
You will always find some on nettle wine's left side;  
Second, different are those who stand at either end,  
But if you would move onward, neither is your friend;  
Third, as you see clearly, all are different size,  
Neither dwarf nor giant holds death in their insides;  
Fourth the second left and the second on the right  
Are twins once you taste them, though different at first sight.

Can you help Hermione identify the bottle that allows Harry to escape and confront the evil-doer?